Вестник ВГУ. Серия: Экономика и управление. 2022. № 4. С. 43–52.
Proceedings of Voronezh State University. Series: Economics and Management. (4), 43–52.

Proceedings of Voronezh State University
Series: Economics and Management

Mathematical and Quantitative Methods

Original article
UDC 338.4
DOI: https://doi.org/10.17308/econ.2022.4/10576
JEL: D24

Model of the production factors of an enterprise as a production function

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Subject. The model of production processes makes it possible to plan the activities of an enterprise based on the availability of production factors and resources. The results of the model can be integrated into the financial model of the enterprise and become part of the financial and production planning process.
Objectives. We proposed an approach to building the model of the production function based on the use of the basic factors of production: labour and capital.
Methodology. The classical representation of the generalised production function was adapted to a specific production process. The main control parameters were the fixed-asset turnover ratio, capital-labour ratio, and labour productivity. A numerical example of the model was provided.
Conclusions. The use of the production function allowed us to describe in detail the internal processes of the enterprise in an accurate and methodologically correct manner. The results of the production model can be integrated into the financial model of the enterprise.

Keywords: Production factor model, enterprise financial model, production function.

For citation: Dzyuba, S. A. (2022). Model of the production factors of an enterprise as a production function. *Proceedings of Voronezh State University. Series: Economics and Management.* (4), 43–52. DOI: https://doi.org/10.17308/econ.2022.4/10576

Introduction

The production factor model of an enterprise has significant common features with the financial model of the enterprise or its business model, though it is conceptually different from them. The financial model of the enterprise primarily reflects the channels of financial relationships between the enterprise stakeholders. In simple cases, they can be described, for example, by the debt-to-equity ratio (Lihachev & Kupriyashina [1]), cash flows and their aggregates (Tikhomirov [2]), including value-based analysis indicators (Kosorukova [3]).

In order to form a financial model, the inputs are the already established cash flows and the ratios between their components. Thus, even using the most basic parameters, it is possible to obtain models that can address fundamental issues (Simonov [4]). To obtain more advanced models, a detailed study of their individual components is required. The top-down modelling principle implies a step-by-step transition from simplified parametrised models to structural decomposition of its components (Mosolova & Biletsky [5]). Thus, if at the stage of parametric modelling we set, for example, the parameter of the ratio of selling costs to revenues as a certain known value, it subsequently becomes a calculated value derived from the revenue and selling costs sub-models. It can be said that the exogenous parameters of the model are to become endogenous upon structural decomposition.

The business model of an enterprise describes the channels of its interactions with the external environment. It is linked to the financial model through the cash flows that are generated in the external environment (Balandina [6]). The business model can be considered as a structural decomposition stage of the financial model, where the latter is disintegrated to the specification of the external environment factors. This also does not eliminate the possibility that the business model can act as an independent model object, as often takes place in actual practice.

The production factor model, following the above logic, can be seen as a decomposition of the internal environment factors, e.g. the models of the utilisation of fuel, electricity, and fixed assets, as explained by Larina &. Petrosov in [7]. Then, the logical connection between the types of models under consideration would be as shown in Figure 1.

In turn, the given relationship should be considered as a special case of a more general system of production factors, proposed by Kleyner [8]. It includes four universal factors α, β, γ, and δ. In the "classical" interpretation, they are associated, respectively, with land (natural resources), capital, management, and labour. The factors have different functions for the external and internal environment:

– in the external environment (business models), their functions are, respectively, development, expansion, densification (intensification), and prolongation (sustainability);

– in the internal environment (production factor models), these are, respectively, integrity, coherence, innovativeness, and safety.

Therefore, it seems crucial that the internal environment model should not simply be a resource model, but rather a production factor model. It logically follows that the core of the model should be the production function, as it is defined in micro- and macroeconomic models.

The aim of this study was to build the production factor model. It is not easy to formulate a general problem for such a model, as it describes the use of the production factors as the output technology. The challenge is that the technologies are highly industry-specific. This is probably why such models are

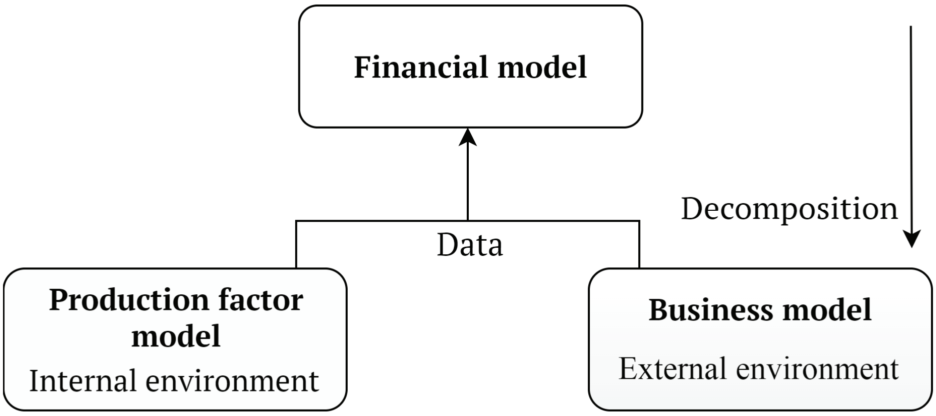

Figure 1. The business model and production factor model as a logical lower level of the financial model of the enterprise

poorly covered in scientific literature, unlike financial and business models. As a prototype, we took trading activities with different sales technologies. First, the workflow is fairly simple. Second, its simplicity should be a benefit for a more accurate representation of the key details.

The paragraph "Materials and methods" describes the general design of the model, while the "Results" section shows its actual implementation and the selection of model parameters using a numerical example.

Materials and methods

Let the sales volume of a trading company for a certain fixed period of time be defined by the production function

$$Y = F(R, K, A, L), \tag{1}$$

Where *R* is the resources: due to the nature of the company, there are no productive resources, only marketing resources aimed at generating a customer flow; *K* is the capital: as a factor of production, capital was represented by the retail space. This space is used to sell different product lines (groups) using different technologies, which require different sets of competencies from the staff; *A* is the technologies: sales techniques for different product groups differ in terms of the specific labour intensity of the transaction; *L* is the labour: selling different product groups requires different competencies.

The model assumes that resources influence capital productivity, and technology influences labour productivity. This approach is a variation of the theoretical constructs for production functions, but is not absolutely universal, concerning the industry specifics. It is fundamentally important that the explicit form of the production function should relate the capital-labour ratio *k* and labour productivity *y* via the fixed-asset turnover ratio *f*:

$$f = \frac{y}{k},\ y = \frac{Y}{L},\ k = \frac{K}{L},\ f = \frac{Y}{L}\frac{L}{K} = \frac{Y}{K}. \tag{2}$$

In this case, the effect of *R* and *A* can be implicit.

For a product (product group) *i* with an average unit price p_i (average purchase amount), the number of sales (transactions) per unit area is defined by a power function:

$$x_i = \beta p_i^{\alpha} + \gamma, \tag{3}$$

where $\alpha\langle 0, \beta\rangle 0, \gamma$ are the parameters to be selected when calibrating the model. Function (3) indicates a negative relationship between the number of transactions and the average purchase amount: the higher the average purchase amount, the fewer transactions there are in the reporting period. The power-law dependence between these values has the same meaning as Zipf's law in economic systems (Dzuba [12]). Then the sales of the product group *i* sold by technology *l* will be

$$Y_i = \frac{p_i x_i K_{l_i} L_l}{K_l}, \tag{4}$$

where K_{l_i} is the part of the area K_l allocated to the product group *i*, L_l/K_l is the number of shop assistants per unit area (shift) using the sales technology *l*.

The meaning of representations (3)-(4) is that for sales by technology *l*, the area K_l is allocated on which different product groups are placed. So, $\sum_i K_{l_i} = K_l$. Product groups were distinguished as they had characteristic average purchase amounts p_i. From (3), we see that the number of transactions does not depend on the technology, but only on the average purchase amount, while the total sales volume (4) depends on the number of employees per shift as the availability of staff with the required competences.

Then, we determined the demand for staff with competencies *l*, i.e., working on the appropriate sales technology:

$$L_l(K_l) = l_0 + \left(\frac{l_1(K_l - \kappa_0)}{\kappa_1}\right)^+, \tag{5}$$

where for the first κ_0 m^2 of the area K_l, l_0 shop assistants are required, and l_1 shop assistants for every additional κ_1 m^2. The positive cut operator $()^+$ means that only positive values of the argument are considered.

Formula (5) imposes a limit on the number of staff based on the sales technology, which does not allow the shift coefficient in (4) to grow indefinitely. If there were no such limit, then in (4) we would have to replace the linear dependence of sales on the number of employees per shift with a functional dependence with a saturation effect (decreasing marginal productivity).

Results

We chose the parameters so that the model gives realistic results. For convenience, all monetary parameters are expressed in thousands of roubles.

Let's assume the trading enterprise has 4 shops M1-M4 with the space structure shown in Table 1.

The product lines were divided into 14 groups (rows in Table 1), for which 6 types of sales techniques were defined: KR1-2, KM1-2, KS1, and KK1. It can also be noted that the largest shop M4 contains the full range of goods, the shop M3 specialises in the high-end segment, and M1 and M2 are in-between.

Let's determine the number of staff according to formula (5), using data from Table 2. It reflects some of the regulatory requirements for the "density" of staff per retail space.

The calculation results are shown in Table 3, rounded to integers. The table reflects the

Table 1

Structure of shop space by product group

Sales technologies (competences)	Shop area					Average purchase amount, thousand RUB
	Total	M1	M2	M3	M4	
KR1	1160	400	360		400	60
KR1	1200	280	240	80	600	70
KR1	700	200		200	300	80
KR2	350			200	150	120
KR2	560			440	120	150
KM1	520	200	120		200	30
KM1	660	200	160		300	60
KM2	260	100		60	100	80
KM2	190			140	50	160
KS1	440	120	160		160	15
KS1	480	160	120		200	20
KS1	160			80	80	25
KK1	440	160	120		160	80
KK1	180	80			100	160
Total	7300	1900	1280	1200	2920	

Table 2

Requirements for the number of staff by formula (5)

Competences	Area, κ_0	Number of shop assistants, l_0	Additional area, κ_1	Additional shop assistants, l_1
KR1	100	4	80	1
KR2	100	3	100	1
KM1	90	2	100	1
KM2	50	2	30	1
KS1	100	4	80	1
KK1	100	3	50	1

Table 3

Calculation of the number of staff by formula (5)

Competences	Area m² per shop assistant	M1	M2	M3	M4
Customer flow		80 %	60 %	40 %	100 %
KR1	41	11	7	4	19
KR2	9			5	4
KM1	13	4	3		6
KM2	12	3		4	5
KS1	22	5	5	4	8
KK1	14	5	3		6
Total	111	28	18	17	48

adjustments for the intensity of the customer flow. One of the shops was taken as 100% and the rest were scaled relative to it.

Next, we defined the parameters of function (3) of the number of transactions x_i dependence on the average purchase amount p_i. The parameters of this function were obtained from the three points $\left(x_i^3,\infty\right)$, $\left(x_i^3,\infty\right)$, and $\left(x_i^3,\infty\right)$ (the upper index indicates the point number):

$$\alpha=\frac{\ln(x_i^2-x_i^3)-\ln(x_i^1-x_i^2)}{\ln p_i^2-\ln p_i^1},\ \beta=\frac{x_i^2-x_i^3}{(p_i^2)^{\alpha}},\ \gamma=x_i^3 \quad (6)$$

Let us formulate the hypothesis (empirically or expertise-based) that turnover increases with an increase in the average purchase amount. It corresponds to the points $(55,15)$, $(7,150)$, and $(0,\infty)$. Indeed, $x_i^1 p_i^1=55\times15=825$, $x_i^2 p_i^2=7\times150=1050$. Then, $\alpha\approx-0{,}8653$, $\beta\approx621{,}3$, $\gamma=0$. The results are shown in Figure 2. With the selected parameters, we were able to simulate the number of transactions for any group of products (average purchase amount).

So, we could use (4) to model the turnover, because we knew the structure of the shops, the number of personnel, and the number of transactions by product group. The results are shown in Table 4.

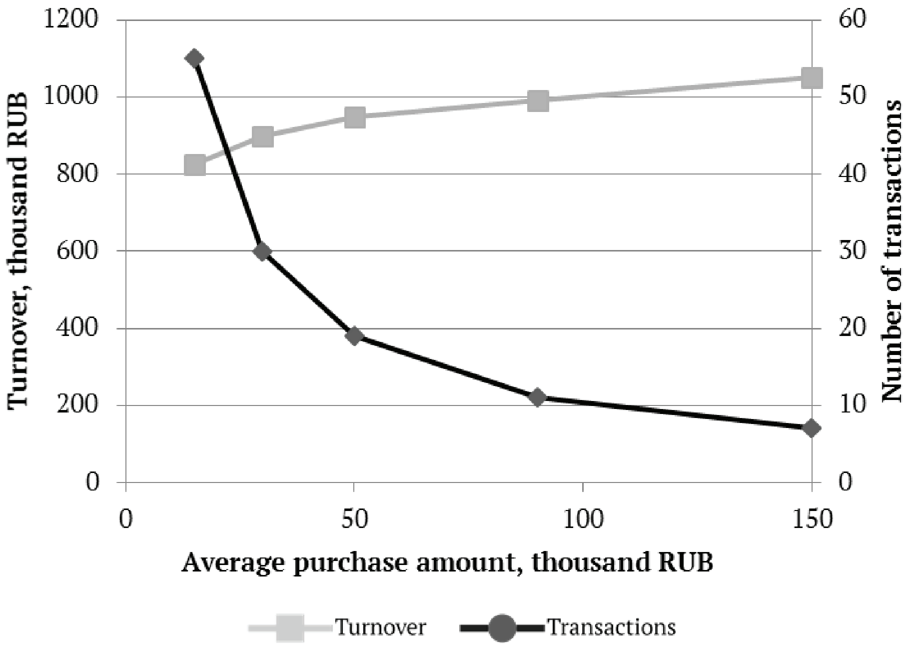

Figure 2. The result of modelling using formula (3): the number of transactions (right axis) and turnover (left axis) as a function of the average purchase amount

Table 4

The number of transactions and turnover calculated by formula (4)

Product group	Transactions				Sales, thousand roubles				
	M1	M2	M3	M4	Total	M1	M2	M3	M4
KR1	79	67		93	14,340	4,740	4,020		5,580
KR1	48	39	16	121	15,680	3,360	2,730	1,120	8,470
KR1	31		35	54	9,600	2,480		2,800	4,320
KR2			13	19	3,840			1,560	2,280
KR2			24	12	5,400			3,600	1,800
KM1	59	38		71	5,040	1,770	1,140		2,130
KM1	32	27		57	6,960	1,920	1,620		3,420
KM2	37		15	41	7,440	2,960		1,200	3,280
KM2			18	11	4,640			2,880	1,760
KS1	118	157		160	6,525	1,770	2,355		2,400
KS1	121	91		155	7,340	2,420	1,820		3,100
KS1			139	51	4,750			3,475	1,275
KK1	41	37		45	9,840	3,280	2,960		3,600
KK1	11			15	4,160	1,760			2,400
Total	577	456	260	905	105,555	26,460	16,645	16,635	45,815

From a formal point of view, we achieved the required result by modelling the sales for different product groups and shops. However, the obtained results may differ significantly from the average actual values. This can be due to both modelling defects, e.g., poor parameter selection, and management failures, which cause the actual results to differ from the normative results. For us, the latter is of particular interest. However, before making any claims to the management, it is necessary to rule out modelling defects as much as possible. For this, we recommend several procedures:

1. Check the fixed-asset turnover ratio, which in this case would be the return per m^2. This value can be obtained by dividing the turnover (sales) from Table 4 by the areas from Table 1. Model defects would be abnormally large deviations from the average.

2. Check the labour productivity. Based on the number of employees from Table 2 and the sales from Table 4 (it must first be grouped according to the technology level), the output per employee can be calculated. Model defects would also be abnormally large deviations from the average.

3. Check the labour intensity of the transactions. Using the number of employees from Table 2 and the number of transactions from Table 4, grouped by the technologies, it is possible to calculate the time cost per transaction. These values can (and should) vary by technique, but not between the shops. The differences between the shops can only depend on the customer flow (see Table 3), which is easily eliminated in the model.

If all abnormalities are eliminated, the model can be considered calibrated. Then, all significant deviations of the average actual values from the model values can be considered management defects.

Results

First of all, it should be stated that the production function is primarily a tool for macroeconomic modelling and, less frequently, for system-wide economic research (Bagrinovsky & Kleyner [9]).

On the other hand, this study has no objective of determining the optimum values of the factors of the production function. An operating enterprise has a fixed amount of capital, and the impact of other factors on its output is difficult to vary or even measure. This allows us to avoid a great deal of microeconomic discussion about optimal cost structures and economies of scale.

Nevertheless, we should note another important feature of the microeconomic approach to production functions. It is the specific elasticity of factor substitution (Kleiner & Piontkovsky [10]). At this level, it is closer to the Leontief type showing the "inflexibility" of the factors, especially with the pronounced industry specifics (Ruzanov [11]).

The approach considered in the study is close to the solution of the problem of developing a digital twin of a production (Makarov et al. [13]). However, it is focused on factor planning rather than the digitisation of technological processes. This is problematic, because empirical studies of the production function are also very challenging (Ackerberg et al. [14]), since in reality we have to deal with far more factors than we use as the arguments of the function (Christensen & Greene [15]). Most of them may be related to the external environment.

On the other hand, the considerable heterogeneity in the representation of the production functions (Gandhi et. al [16]) is in line with the idea promoted in this study. We propose, in fact, an expertise-based construction of the production function for the utilitarian purposes of production and financial planning of the current activities of the enterprise. An important condition for "restricting" the heterogeneity is to limit the expert's perceptions to formal representations of the production function based on the basic factors and to check key performance indicators.

Conclusions

The production factor model details the internal environment of an enterprise. It is constructed as a production function of the main factors, which are taken as a generalised universal representation by Kleyner [8].

We considered in detail the production function of the trading enterprise. As the capital factor, we took the retail space, which can be either rented or owned. Labour is represented by a set of various sales technologies, differing in terms of labour intensity and the requirements for staff competences. Therefore, it also involves the technology factor. It is most convenient to consider the customer flow as a limited resource factor.

The proposed model can be adapted to the digital trading enterprise with no fundamental changes. In such a case, warehousing and logistics capacities are considered instead of trading capacities.

The model has a system of internal parameter control, based on the analysis of significant local deviations of the factor performance indicators. The parameters of the model can be considered calibrated, if there are no such deviations. In this form, it can be integrated into the financial model of the enterprise. It can also be used independently to analyse the quality of management of internal processes based on the deviation of the average actual parameters from the model values.

Acknowledgements

The author is grateful to Elena A. Tyurina for her helpful and constructive remarks during the discussion of this study.

Conflict of interests

The author declares the absence of obvious and potential conflicts of interest related to the publication of this article.

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Received: 31.10.2022
Accepted: 30.11.2022

Вестник ВГУ. Серия: Экономика и управление. 2022. № 4. С. 43–52.
Proceedings of Voronezh State University. Series: Economics and Management. (4), 43–52.

Вестник Воронежского государственного университета
Серия: Экономика и управление

Математические и инструментальные методы экономики

Научная статья
УДК 338.4
DOI: https://doi.org/10.17308/econ.2022.4/10576
JEL: D24

Модель факторов производства предприятия как производственной функции

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Предмет. Модель производственных процессов позволяет осуществлять планирование деятельности предприятия исходя из наличия факторов производства и ресурсов. Результаты модели могут быть интегрированы в финансовую модель предприятия и стать частью процесса финансового и производственного планирования.
Цели. Предлагается подход к построению модели производственной функции, исходя из способов использования базовых факторов производства: труда и капитала.
Методология. Классическое представление обобщенной производственной функции адаптируется под конкретный производственный процесс. В качестве основных контрольных параметров берутся фондоотдача, фондовооруженность и производительность труда. Представлен численный пример модели.
Выводы. Использование производственной функции позволяет корректно и методологически правильно детализировать процессы внутренней среды предприятия. Результаты производственной модели могут быть интегрированы в финансовую модель предприятия.

Ключевые слова: модель факторов производства, финансовая модель предприятия, производственная функция.

Для цитирования: *Дзюба С. А.* Модель факторов производства предприятия как производственной функции // Вестник Воронежского государственного университета. Серия: Экономика и управление. 2022. № 4. С. 43–52. DOI: https://doi.org/10.17308/econ.2022.4/10576

Благодарности

Автор благодарит Тюрину Елену Александровну за полезные и конструктивные замечания при обсуждении настоящей работы.

Конфликт интересов

Автор декларирует отсутствие явных и потенциальных конфликтов интересов, связанных с публикацией настоящей статьи.

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Поступила в редакцию: 31.10.2022
Подписана в печать: 30.11.2022