



## Mathematical and Quantitative Methods

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# Dynamics of the formation of social groups and adjustment of characteristics of individuals in the process of them uniting into groups

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**Subject.** This article discusses the agent-based model describing the process when two or more individuals meet for the first time and form a small social group based on interests. It is assumed that when they get to know each other, they exchange information about certain phenomena of the surrounding world and their attitudes to them. The article describes a study which involved simulation of the process of how the significance of these phenomena in their understanding of the surrounding world by each individual changes as a result of such interactions.

**Purpose.** To determine the possibility of the formation of interest groups in a random sample of individuals and the nature of changes in their attitudes to the world as a result of several consecutive interactions between them within the paradigm of agent-based models.

**Results.** The method of hierarchical cluster analysis revealed stable interest groups which were formed within a random sample of individuals. A series of interactions between the participants of these groups significantly expanded the range of the participants' ideas about the surrounding world and their confidence that their attitudes to the phenomena of the surrounding world were right.

**Conclusions.** It was shown that groups of individuals with completely opposite characteristics can emerge even in a random sample of individuals. What is more, the participants of each group will have similar ideas about the surrounding world. It was also shown that repeated interactions between the individuals within these groups boost their confidence about their attitudes to certain phenomena of the surrounding world. It was noted that currently the Internet is an important platform for such interactions.

**Keywords:** agent-based models, social groups, principal components method, personality structure, thesaurus.


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### Introduction

This article describes the agent-based model (hereinafter referred to as the ABM), which simulates a situation when two or more people meet for the first

time and form an informal interest group (hereinafter referred to as the ATM model), and a series of calculations, which were made within this model.

It is challenging to find a place for this model in the framework of the theory of small social groups (Katz et al., 1956; Andreeva, 2021; Mejzhis &

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Pochebut, 2010; Sokolov, 2002). In particular, the concept of “small group” has many definitions based on the size of the group and the time of its existence, on its structural features, the nature of the relationships within the group, and its role in the society as a whole. For example, according to the definition given by G. M. Andreeva “...a small group is a group in which *social relations* manifest themselves in the form of direct personal contacts” (Andreeva, 2021, C. 184). According to the definition of the authors of the book (Mejzhis & Pochebut, 2010, section 13.1) “...a *social group* is a community of people separated from an integral social system by a certain characteristic.” There are many more definitions of the concept of a small group, however, the one that fits the best the sample of individuals formed by the ATM model is the definition given by Bales (Bales, 1962): “...a small group can be defined as a number of people interacting with each other during one or more meeting(s)” (Mejzhis & Pochebut, 2010, section 13.1), where according to the ATM model an “interaction” is an exchange of opinions among the members of the group aimed at establishing an increasingly strong consensus among the members of the group with regard to their attitude towards certain phenomena of the surrounding world.

An important motive which led to the creation of the ATM model within the ABM paradigm was the idea that a proper socio-psychological study of a particular small group means “interfering” with its routine, which can disrupt the structure of relationships between group members and result in distortions in the way the things are within the group. A detailed description of these problems can be found in (Andreeva, 2021, pp. 62–66, Ch. 19). At the same time, a computer model allows identifying the patterns of emergence and development of a small group as is, though, of course, with certain limitations with regard to certain characteristics of the members of the group and the group as a whole. Such modelling offers opportunities which are quite difficult to create in a real socio-psychological experiment. For example, in the ATM model, a lot of attention is paid to the process describing how groups of individuals emerge, which is never described when studying groups in applied, and especially in theoretical research.

In conclusion, we would like to say that the possibility of creating such a model appeared due to

the emergence and widespread use of agent-based modelling<sup>1</sup>. The theoretical description of the ATM model is given in the articles (Andrukovich, 2021; Andrukovich, 2018). This article discusses the specific application of the ATM model to analyse the process of how groups of individuals emerge from a random sample of individuals and how consensus is established between the members of these groups with regard to their attitude to the phenomena of the surrounding world.

### Research Methods

In this model, each individual (hereinafter referred to as II) is described by a certain combination of ideas and knowledge about the surrounding world, whose elements and phenomena are called “*concepts*” and their sum-total is called the “*thesaurus*” of the individual. It is believed that concepts known to a given individual are a subset of a set of all concepts describing the phenomena of the surrounding world. These general concepts are not ordered in any way, i.e. they do not have any assessments on the scales of “evil – good”, “right – wrong”, “good – bad”, etc. However, each II might have definite attitudes to the concepts known by them and these attitudes can be positive, negative, or indifferent. What is more, since concepts known by II have different significance for them within the structure of their ideas about the surrounding world, each concept in the II’s thesaurus has its “*weight*”. All concepts known by II, their attitudes to these concepts, and the weights of these concepts are called the *personality structure* of this II.

The II’s thesaurus and their personality structure are used to form five pairs of generalised characteristics of the II calculated for these concepts and weights of concepts. Firstly, this is the number of concepts known to the II (and the sum of weights of these concepts) and their proportion in the total number of concepts (or the total sum of weights), and, secondly, the sum of concepts (and their weights) to which they have a positive, negative, or indifferent attitude. These aggregate estimates for each II are used to calculate the *coefficient of differentiation* of

<sup>1</sup> Within the framework of agent-based models, a large number of models have been created that simulate the behaviour of agents in a particular field of activity. Among important works dedicated to this topic are, in particular, the following publications: (Bonabeau, 2002; Bakhtizin, 2008; Guts et al., 2000; Istratov, 2006, 2018; Makarov, 2006; Makarov & Bakhtizin, 2013; Mes & Gerrits, 2019).

their personality structure. This coefficient is equal to the proportion of concepts (and the proportion of the total weight of concepts) to which this II is not indifferent relative to the total number (and total weight) of concepts known to them.

In addition to these generalised characteristics which were built based on the personality structure of the II, we also introduced for each II an indicator of the level of their *social engagement*, which is set by the value of a random variable distributed over a half-interval (0.1). It is obvious that according to its construction principle this value is exogenous in relation to the personality structure of the II. Small values of the II's characteristic indicate that they are passive in relation to the surrounding world, while high values indicate their desire to share their views on the surrounding world with other IIs.

The above-given principles of describing IIs are used to create a sample, in which the size of the thesaurus of each II, the nature of their attitudes to the surrounding world, and the degree of their social engagement are created using a random data generator. The distribution of these characteristics over the sample is given a priori. The weights of concepts are considered equal, with a single value at the beginning of the work of the ATM model. Further, this sample is used to simulate the process of how groups of IIs emerge, how they exchange information about the phenomena of the surrounding world known to them and about their attitudes to them. This process involves adjustment of the personality structures of IIs when they are brought together, when an II can learn about new concepts which had been unknown to them before this interaction, or when an II can change their attitude to certain concepts which had already been known to them or the degree of their confidence in relation to this or that concept, which in this model imitates the weight of the concept. Such adjustment is the core of the ATM model since it allows tracing the dynamics of changes in the personality structures of IIs as a result of their interaction with each other, as well as when an II joins an existing group of IIs.

II groups can be formed and their personality structures can be adjusted several times. This process simulates a gradual change in these characteristics of IIs after repeated interactions between them. To simulate this process, in each consecutive cycle of interactions, we take those characteristics of IIs that manifested themselves at the previous stages of their

interactions as their basic personality structure and the values of their social engagement.

The ATM model was implemented as a set of programs created, as has been mentioned above, within the framework of the theory of agent-based models. This article analyses how this model works for a sample of IIs whose characteristics are determined using a random data generator, i.e. for a random sample of IIs, including the construction of their groups by hierarchical cluster analysis (hereinafter referred to as HCA) (Braddock, 1958; Yenukov, 1989; Jambu, 1988; Kendall & Stewart, 1973; Terekhin, 1973) and adjustment of their personality structures as a result of their repeated interactions with each other.

## Results

**Setting the original sample.** In this study, we used a sample of 100 IIs in which the personality structure of each individual was a subset of a total set of 50 concepts. Three groups of parameters were introduced into the ATM model to configure a specific sample of IIs (with the exception of the above two parameters). These three groups of parameters included, firstly, setting the distribution of the size of IIs' thesauruses, secondly, setting proportions of IIs' positive, negative, and indifferent attitudes to concepts, and, thirdly, setting the distribution of the level of IIs' social engagement.

The calculations presented below show that the sizes of IIs' thesauruses had an upper limit of 90 % of the total number of concepts and a lower limit of 15 % of the same number of concepts<sup>2</sup>. The proportion of IIs with large thesauruses (between 0.67 and 0.9 of the total) was a small fraction of the sample of IIs (15 %), the proportion of IIs with small thesauruses (between 15 % and 1/3) was 35 % of all IIs, IIs with an average size of thesaurus was a half of the sample. On average, the positive, indifferent, and negative attitudes of IIs to concepts were distributed equally throughout the sample (i.e. one third for each). The levels of their social engagement obtained by the random data generator were also evenly distributed within a half-interval (0.1)<sup>3</sup>.

<sup>2</sup> The upper limit was introduced to rule out "know-it-alls" among IIs, and the lower limit was given to eliminate from the sample individuals who knew just a couple of concepts, and especially "zero" individuals.

<sup>3</sup> That means that an II's social engagement could not be zero.

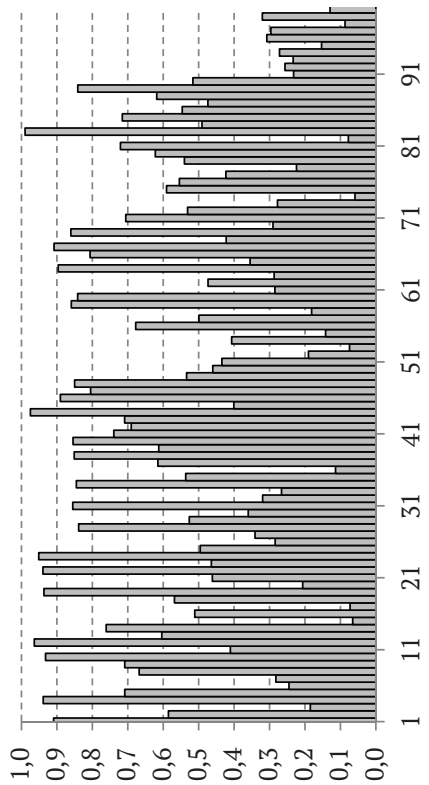


Fig. 1b. Levels of social engagement of individuals

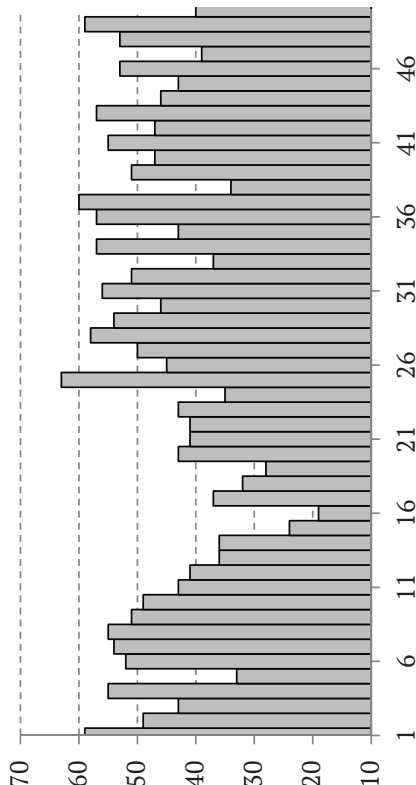


Fig. 1d. Number of individuals who know the concept

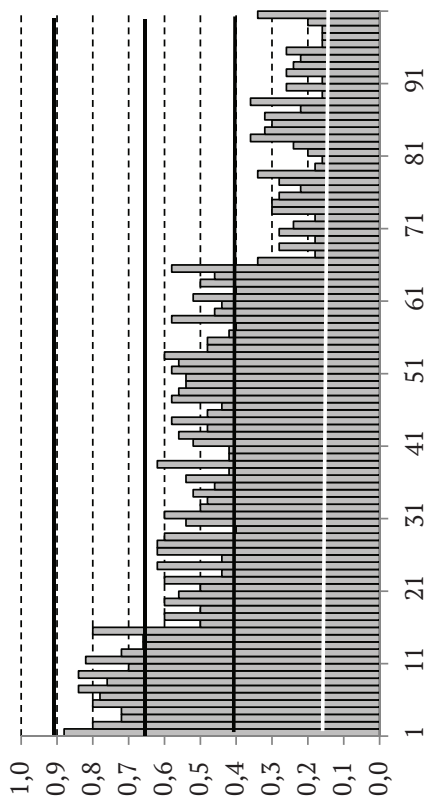


Fig. 1a. Distribution of proportions of thesauruses by individuals

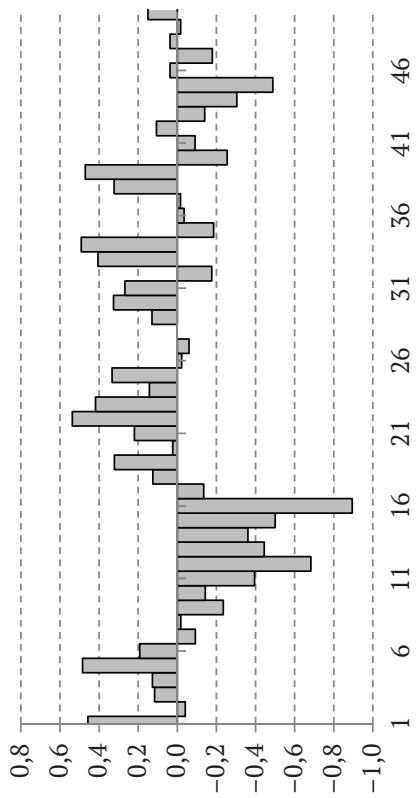


Fig. 1c. Average attitude of individuals to the concept

Fig. 1. Graphical description of the original sample (the horizontal axis in Figs. 1a and 1b stands for the number of the II, and in Figs. 1c and 1d it stands for the numbers of concepts) (calculated by the author)



Fig. 1a–1d show a graphical description of the sample obtained at the specified values of the parameters.

**Formula of proximity of personality structures of IIs.** A sample of IIs created in this way was used to simulate the process of how IIs meet each other for the first time and how groups of them formed by means of the HCA method. For this purpose, a special *measure of proximity* of the personality structures of two IIs was introduced into the ATM model (hereinafter  $S_1$  and  $S_2$ ). It was based on the contingency table which was built for the personality structures of a pair of IIs (Table 1). In this table,  $n^{++}$  stands for the number of concepts to which both individuals had a positive attitude,  $n^{0+}$  is the number of concepts to which individual  $S_1$  was indifferent and the attitude of individual  $S_2$  was positive;  $n^{-+}$  is the number of concepts to which the attitude of individual  $S_1$  was negative, and of individual  $S_2$  was positive, etc.  $n_2^+$ ,  $n_1^+$ , etc. are the number of concepts to which the attitude of individual  $S_2$  was positive while individual  $S_1$  was ignorant of them (and similarly  $n_1^+$ ,  $n_1^0$ , etc. are the number of concepts to which the attitude of individual  $S_1$  was positive while individual  $S_2$  was ignorant of them).  $NH$  is the total number of concepts.

Table 1

Number of concepts with similar or different attitudes to them of individuals  $S_1$  and  $S_2$

Individual $S_2$	Individual $S_1$				Total
	+1	0	-1	Ignorant	
+1	$n^{++}$	$n^{0+}$	$n^{-+}$	$N_2^+$	$N_2^+$
0	$n^{+0}$	$n^{00}$	$n^{-0}$	$n_2^0$	$N_2^0$
-1	$n^{+-}$	$n^{0-}$	$n^{- -}$	$n_2^-$	$n_1^0$
Ignorant	$n_1^0$	$n_1^0$	$n_1^-$	$m_{12}$	$N_2$
Total	$N_1^+$	$N_1^0$	$N_1^-$	$N_1$	$NH$

The degree of proximity  $P$  of individuals  $S_1$  and  $S_2$  in the designations in Table 1 is calculated by the formulas

$$P = p_{ik} + \nu b_{ik}, \tag{1}$$

where

$$p_{ik} = (n^{++} + \delta n^{00} + n^{- -}) - (n^{-+} + n^{+-}), \tag{2}$$

and

$$b_{ik} = n^{0+} + n^{0-} + n^{-0}. \tag{3}$$

Parameters  $\delta$  and  $\nu$  allow changing the significance for assessing the level of proximity of those concepts to which the attitude of one of

the individuals (parameter  $\nu$ ) or both individuals (parameter  $\delta$ ) was neutral<sup>4</sup>. As can be seen from these formulas, concepts that at least one of the individuals within a pair was ignorant of were not involved in calculating the level of proximity. The measure of proximity given in formulas (1)–(3) for the number of concepts can also be applied when weights of concepts are necessary to calculate proximity. For this purpose, Table 1 should include the sums of weights of the corresponding concepts instead of the number of concepts  $n^{++}$ ,  $n^{0+}$ , etc.

It is believed that at sufficiently high values of the measure of proximity between a pair of IIs, a *contact* occurs, as a result of which they get the opportunity to exchange certain information with each other. In this calculation, concepts with indifferent attitudes of both IIs had the proximity assessment with a weight of  $\delta = 0.75$ , while in pairs of individuals, one of whom had a neutral attitude to this concept, and the other had a definite attitude, the proximity assessment had a weight of  $\nu = 0.25$ <sup>5</sup>.

**Classification of IIs.** In total, three cycles of calculations were carried out, each of which determined groups of IIs and the nature of changes in their personality structures. For example, the first calculation performed for the initial sample of IIs revealed six groups of IIs. One of them consisted of 16 IIs (hereinafter, Gr1), two groups consisted of 13 IIs (hereinafter, Gr2 and Gr3), two groups had two IIs each, and one more group consisted of three IIs<sup>6</sup>. 51 IIs (Gr4) were not included in any of these groups. Their level of proximity to the resulting groups was lower than the minimum level of proximity specified in the calculation<sup>7</sup>.

<sup>4</sup> Therefore, it is worth mentioning that, considering the form of formula (1.1),  $P$ , the value of proximity, can also be negative. This is quite natural, since the concept of “negative proximity”, similar to negative correlation, assesses the level of antagonism between a particular pair of individuals.

<sup>5</sup> Before the analysis, we would like to note that, since the sums of the weights of concepts are measured in conditional units depending on their initial, conditional weight which has been taken as a unit in the ATM model, hereinafter, to indicate units of measurement of a particular characteristic of an II without introducing the abbreviation CU (conditional units), which is too strongly associated with completely different measurements, we will use the term “special units”. To make the text shorter, we will replace it with the abbreviation SU.

<sup>6</sup> Such small groups confirm the validity of the chosen HCA method since it was impossible to predict them considering the fact that the original sample was random. It should also be noted that in the future we are not going to consider the characteristics of these small groups.

<sup>7</sup> This level was determined using the method described in (Andrukovich, 2021). In this calculation, it was 16 SU.

As a result of the second cycle of calculations, when the minimum level of proximity was equal to 19 SU and the personality structures of IIs had already been adjusted during the first cycle, five groups of IIs were formed. The composition of the major groups did not change, and the composition of one of the smaller groups remained the same, the second did not form at all, and the group consisting of three IIs transformed into a group of two IIs since one II had left this group. 54 IIs did not join any of these groups. Finally, during the third cycle of calculations with a minimum level of proximity equal to 23 SU, six groups of IIs were formed. Two of them, Gr2 and Gr3, remained the same, while five IIs left group Gr1 and formed two groups consisting of two and three IIs. Another group of two people remained unchanged. 56 IIs did not join any of the groups. Thus, two groups, Gr2 and Gr3, remained unchanged throughout all three cycles of calculation. It should also be noted that the sequence in which IIs joined each of these groups varied from cycle to cycle. Yet, the resulting composition of these two groups was the same with a slight change during the third cycle for group Gr1.

To analyse the structure of ratios between these groups in all three calculations, we used the method of principal components (hereinafter referred to as PC) (Jolliffe, 2002; Pearson, 1901; Andrukovich, 1973). The matrix for initial data used to calculate loads and values of factors for IIs were the values for six generalised characteristics of IIs:

1) the sum of the weights of all concepts known to the II (amount of knowledge);

2) the sum of the weights of concepts with a negative attitude to them of the II;

3) the sum of weights of concepts that were the II was indifferent to;

4) the sum of weights of concepts with a positive attitude to them of the II;

5) the coefficient of differentiation of the II's personality structure calculated with due account of the weight of concepts known to this II;

6) the level of social engagement of the II.

Values of PC loads at these indicators, their average values, and values of standard deviations during the first calculation are given in Table 2. As can be seen from the data in the table, the main difference between IIs manifested itself in the opposition of IIs with a high level of differentiation of opinions about the surrounding world and IIs indifferent to the most phenomena of the world known to them. The second PC highly assessed IIs who had a good knowledge about the surrounding world, had predominantly negative attitude to it, and were characterised by a fairly high social engagement, as opposed to IIs who had a smaller amount of knowledge, were socially apathetic but had a positive rather than negative opinion about the concepts known to them. Finally, the third PC clarified the differences between IIs revealed during the first two PCs and divided individuals into those who had mainly positive attitudes to the concepts and those whose attitudes were mainly negative. Similar PC calculations carried out for the generalised characteristics of IIs during the second and third cycles of calculations revealed the same factors whose load vector angles from those given in Table 2 did not exceed 0.2 degrees.

Table 2

*Loads of the principal components at the six generalised characteristics of IIs and their average values and standard deviations*

Abbreviated indicators	Loads of principal components			Average values	Standard deviations
	U1	U2	U3		
Amount of knowledge	0.122	0.001	<b>0.797</b>	0.705	0.383
Negative attitude	0.349	<b>-0.661</b>	0.165	8.169	11.586
Indifferent attitude	<b>-0.567</b>	0.004	0.379	9.610	11.567
Positive attitude	0.342	<b>0.718</b>	0.256	8.404	10.725
Differentiation	<b>0.634</b>	0.028	-0.082	0.628	0.375
Social engagement	0.150	-0.215	0.348	0.525	0.275
PC variance	<i>2.283</i>	<i>1.342</i>	<i>1.289</i>	-	-

*Note.* The total variance of these PCs was 81.9 % of the total variance (calculated by the author).

To correlate the nature of these three factors with the configuration of II groups, Fig. 2 shows the location of the obtained groups based on the results of the first (left) and third (right) calculation cycles in the projection on three two-dimensional fields constructed for each pair of the first three

corresponding PCs (Andrukovich, 1977; Zinoviev, 2000). In these diagrams, IIs from group Gr1 are indicated as diamonds, from group Gr2 as circles, and from group Gr3 as squares. IIs that were not included in these three groups (group Gr4) are indicated as triangles in these projections.

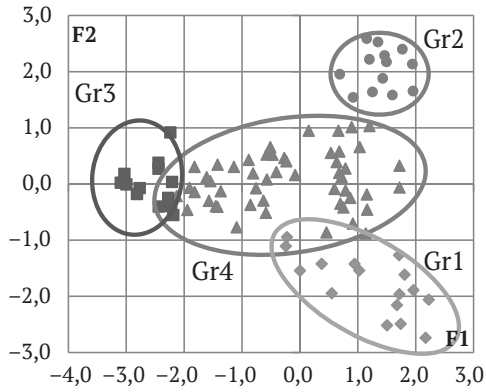


Fig. 2a. Projection of the sample on the plane of the first two principal components after the first cycle of contacts between the IIs

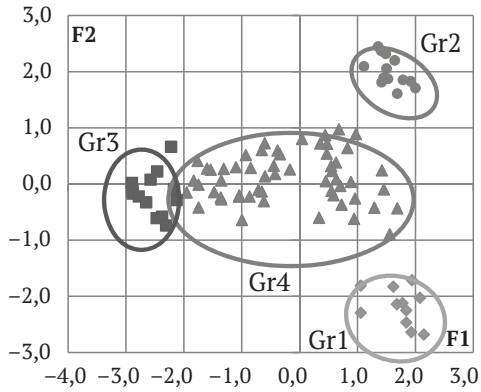


Fig. 2b. Projection of the sample on the plane of the first two principal components after the third cycle of contacts between the IIs

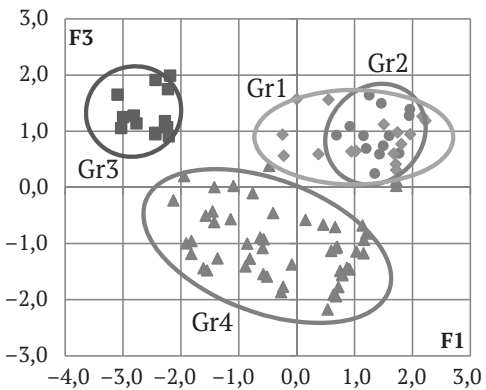


Fig. 2c. Projection of the sample on the plane of the first and third PCs after the first cycle of contacts between the IIs

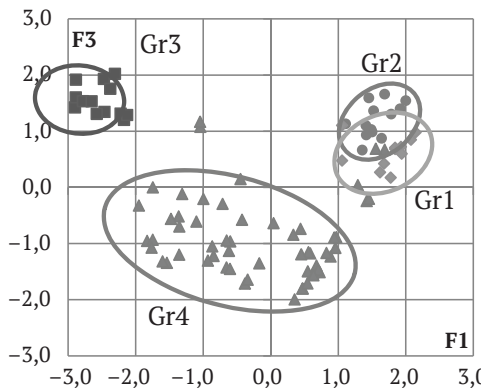


Fig. 2d. Projection of the sample on the plane of the first and third PCs after the third cycle of contacts between the IIs

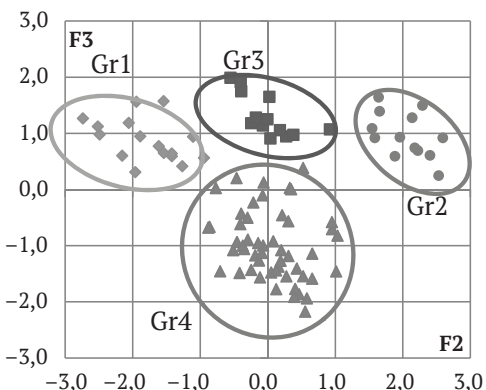


Fig. 2e. Projection of the sample on the plane of the second and third PCs after the first cycle of contacts between the IIs

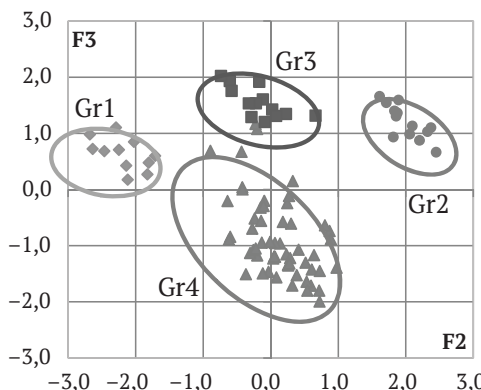


Fig. 2f. Projection of the sample on the plane of the second and third PCs after the third cycle of contacts between the IIs

Fig. 2. Projections of II groups on three pairs of principal components (Fig. 2a, 2c, and 2e) after the first and third calculation cycles (Fig. 2b, 2d, and 2f) (calculated by the author)

From the diagrams presented in the left and right parts of Fig. 2, it can be seen that, firstly, II groups differ from each other in their location on these projections (with the exception of the projection of groups Gr1 and Gr2 on the plane of the first and third PC) and, secondly, that the structure of their location relative to each other during the three calculation cycles did not change and only became more definite.

Comparison of the location of these groups in Fig. 2a–2f with the values of PC loads in Table 2 shows that for the first PC, individuals from groups Gr1 and Gr2 were characterised by a high level of differentiation in their attitudes towards the phenomena of the surrounding world. What is more, the second PC showed that the attitudes to the world among the individuals from group Gr2 was generally positive, while among the individuals from group Gr1 it was negative. At the same time, individuals from group Gr3 were generally indifferent to certain phenomena of the surrounding world (Fig. 2a and 2b). It is also characteristic that IIs from group Gr4 are located in the centre, which means that the attitudes of these individuals to the surrounding world was less definite than those of the IIs from the three main groups. The location of the groups in the projection on the first and third PCs (Fig. 2c and 2d) shows that the amount of knowledge of the “outsider” IIs was significantly smaller than that of individuals from the three main groups. What is more, the coinciding location on this projection of groups Gr1 and Gr2 is due to the fact that for the third PC, the difference in the direction of their attitudes to the world was not taken into account<sup>8</sup>.

The most specific characteristic of the differences between all four groups of IIs is their location in the projection on the second and third PC (Fig. 2e and 2f). Indeed, the three main groups can be found at about the same level in terms of their size of thesauruses. What is more, they form a sequence (from left to right) of personality structures: from a negative attitude of the individuals in group Gr1, through an indifferent attitude to the phenomena of the external world in group Gr3, to a positive attitude to this world in group Gr2. The group of “outsider” IIs is located, in

<sup>8</sup> Both corresponding PC loads are positive and equal to 0.165 and 0.256 (Table 2, columns U3).

terms of the size of their thesauruses, below these groups and occupies a neutral position relative to the conventional scale of “negative-positive” attitude to the surrounding world.

It is worth saying, that a comparison of the location of these groups after the third cycle to their location after the first cycle of calculations showed that the “cohesiveness” of IIs within these groups had increased significantly. This was also confirmed by the data from Table 3, which shows relative intra-group distances between individuals in all of these groups<sup>9</sup>. Thus, the intra-group distance for group Gr1 after a slight increase during the second cycle of calculations decreased by half after the third cycle relative to the first contacts between the group members<sup>10</sup>. The same two-fold decrease in intra-group distances, but with a monotonous change, took place between the individuals of group Gr2, and to a smaller degree the individuals from group Gr3. It is quite obvious that the level of the average intra-group distance between the IIs from group Gr4, due to the lack of contacts between them (and with the individuals of the other groups), did not change over all of the three cycles.

Table 3  
*Relative intra-group distances between individuals from the three groups and for “outsider” IIs (Gr4 group) after three cycles of contacts*

No. of iteration	Relative intra-group distances (in SU)			
	Gr1	Gr2	Gr3	Gr4
1	0.108	0.060	0.062	0.257
2	0.117	0.044	0.048	0.268
3	0.055	0.029	0.040	0.263

*Note.* Calculated by the author.

**Analysis of group characteristics.** Let us now consider how the properties of these groups changed over the three cycles of II’s contacts with each other (Table 4). Before analysing the data from this table, we would like to note that the comparison of II groups by the sum of weights of concepts within the interpretation adopted in the ATM model shows the degree of confidence

<sup>9</sup> Due to a significant increase in the total weight of the concepts that characterised individuals during the first, second, and third iteration of contacts between them, the intra-group distance was calculated relative to the total weight.

<sup>10</sup> This happened, in particular, due to the exclusion of five IIs from this group, as we have already mentioned above.



The average values of the total weights of the concepts for the three major groups of IIs obtained during their classification, and for the "outsider" IIs (Gr4)

Name of indicators	First cycle				Third cycle			
	Group name				Group name			
	Gr1	Gr2	Gr3	Gr4	Gr1	Gr2	Gr3	Gr4
Amount of knowledge	36.7	36.5	37.5	16.1	49.1	46.7	47.7	17.2
Negative attitude	<b>29.3</b>	0.62	0.97	3.92	<b>43.8</b>	0.49	1.27	4.63
Indifferent attitude	5.66	3.95	<b>34.8</b>	6.69	2.98	3.42	<b>44.4</b>	6.78
Positive attitude	1.74	<b>31.9</b>	1.71	5.53	2.36	<b>42.8</b>	2.00	5.77
Differentiation	0.83	0.90	0.07	0.60	0.94	0.93	0.07	0.60
Social engagement	0.62	0.54	0.43	0.49	0.66	0.55	0.43	0.51
Number of individuals	16	13	13	51	11	13	13	56

Note. In special units (SE) (calculated by the author).

of the group about their point of view with regard to the surrounding world and its readiness to take certain actions to implement it.

The first thing that can be noted here is practically equal average values of the sums of the weights of the concepts known to IIs in all three major groups, both after the first and the third cycle of contacts between them. That means that the amount of knowledge of the members of these groups was quite large (about 70 % of all concepts for each of these groups after the first cycle of contacts and more than 80 % after the third cycle) and they were equally convicted that the attitude to the surrounding world characteristic of their group was right. However, the direction and the type of their aspirations (or a lack thereof) to change the external world were fundamentally different.

For example, if we analyse the generalised characteristics of the groups after the first cycle of IIs' contacts, we can see that IIs from group Gr2 had a positive attitude to the world with the weight of the corresponding concepts of 88 % of the total weight of 36.5 SU, while the average number of concepts to which IIs from groups Gr1 and Gr3 had a positive attitude was only 4.8 and 4.7 % of their total weight. As for the members of group Gr1, they had quite a definite negative attitude towards the surrounding world: the weight of such concepts in this group was 80 % of the total weight of 36.7 SU, while IIs from group Gr3 were practically indifferent to the world (92.8 % of the

total weight of the concepts they know). In other words, the members of group Gr3, given that the social engagement of this group was the lowest of the three groups (0.43 SU), were absolutely convinced that they did not want to change the world around them and, figuratively speaking, they "didn't give a damn" about it, while the members of group Gr1 believed that this world should be changed because they did not like it very much<sup>11</sup>. These characteristics became even more pronounced after the third cycle of contacts (Table 4, right part).

As for IIs that did not join any of the groups, the reason for their not belonging to any of the groups was the low amount of knowledge: 16.1 and 17.2 SU after the first and third cycles, respectively. IIs with the smallest sizes of thesauruses (Fig. 1) find it much more difficult to establish a higher level of proximity with IIs with larger thesauruses (at least to the extent that it is used in the ATM model) than the level of proximity between IIs with large or medium thesauruses and large weights of concepts known to them.

<sup>11</sup> As we remember, in the ATM model, the initial sample of concepts does not, generally speaking, have any attitudinal meaning. Therefore, a negative attitude to concepts is not negative from the point of view of ethical, moral, religious or any other ideas about the surrounding world. In particular, a positive attitude to negative, from a social point of view, concepts can be, from this point of view, a "bad" and unacceptable attitude. However, an indifferent attitude of IIs from group Gr3 to the surrounding world is a negative characteristic.

Considering the differences between the groups with regard to the degree of differentiation of the personality structures of their participants, the highest values in both cases were characteristic of IIs from groups Gr1 and Gr2 (0.83 and 0.94 and 0.90 and 0.93, respectively) and the lowest of group Gr3 (0.07 in both cases). In terms of the level of social engagement, the highest values after the first and second cycles of contacts were characteristic of groups Gr1 and Gr2 (0.62 and 0.54, respectively), and the lowest of group Gr3. That, given its indifferent attitude to the outside world, allows us to define this group as a passive layer in the community of all individuals from entire original sample. It should be noted in conclusion that the above figures fully confirm the general descriptions of the groups that were given when analysing their location in Fig. 2a–2f.

**Changes in personality structures of IIs during their contacts**<sup>12</sup>. When there is a contact between two IIs or when an II joins a group which has been formed earlier, the ATM model assumes three possible situations:

- 1) a new concept with a certain weight is included in the personality structure of the II;
- 2) the II changes their attitude to the concept and its weight if the concept is already known to the II;
- 3) no changes take place in the personality structure of the II.

In the first case, most often a new concept is included in the personality structure of the II along with the attitude to this concept of the II (or a group of IIs) with whom this II came into contact. What is more, the concept is included with a certain, possibly adjusted, weight of this concept<sup>13</sup>. However, in some cases, at a certain value of a random number and the corresponding threshold for it set before the beginning of the calculation, an II recipient can develop an indifferent attitude to this concept taking it as some new knowledge, about which they have not

yet formed an opinion.

In the second case, if the opinions of both IIs or an II and a group of IIs coincide completely, the weight of the corresponding concept increases. However, if one of this pair of subjects has an indifferent attitude to this concept, the II recipient may retain their attitude to this concept (with the same or a smaller weight) or may change for another attitude also with a smaller weight. If the opinions of both IIs, or an II and a group of IIs with regard to this concept are opposite, the choice of the attitude to this concept of each of the IIs participating in the contact depends on the ratio of the levels of their social engagement. Either it remains the same, but probably with a smaller weight, if the social engagement of the first II is higher than the social engagement of the second II (or a group of IIs) or it changes to an indifferent attitude in the opposite case.

Let us move on to the analysis of the nature of the adjustment of the personality structures of IIs. In Fig. 3 you can see graphs of differences in the total weight of concepts, changes in the weights of the three types of IIs' attitudes to the concepts known to them, the coefficient of differentiation of their opinions about these concepts and their social engagement for those IIs whose personality structures were adjusted in the process when II groups formed over the three cycles of contacts<sup>14</sup>. As can be seen from these diagrams, there were significant changes in the personality structures of IIs who joined a certain group. For example, the sum of weights of all concepts known to II increased for all IIs who joined a certain group even after the first cycle of contacts (by an average of 5.6 SU within a range from 1.5 to 14.9 SU). After the third cycle it increased by an average of 16.2 SU within a range from 8.3 to 23.2 SU<sup>15</sup>.

24 out of 42 IIs changed their negative attitude to concepts after the first cycle of contacts (however, as can be seen from Fig. 3d, a decrease in

<sup>12</sup> The choice of appropriate rules for the adjustment of the personality structures of IIs was largely based on such publications as (Andreeva, 2021; Guts et al., 2000; Mejzhis & Pochebutr, 2010). Detailed description of all relevant conditions is given in (Andrukovich, 2021).

<sup>13</sup> In some cases, the weight of the concept may decrease depending on the values of certain parameters which were set a priori. For details, see (Andrukovich, 2021).

<sup>14</sup> Since IIs with a small amount of knowledge did not join any group, the graphs in these diagrams are limited on the right by the last ordinal number of the II whose personality structure was adjusted. It should also be noted that all estimates of changes in the characteristics of individuals provided below were given only for IIs who joined groups Gr1, Gr2, and Gr3, the number of which in the groups after the first cycle was 42 IIs, and after the third cycle 37 IIs.

<sup>15</sup> This diagram has no negative values since there is no concept of "forgetting" in the ATM model.

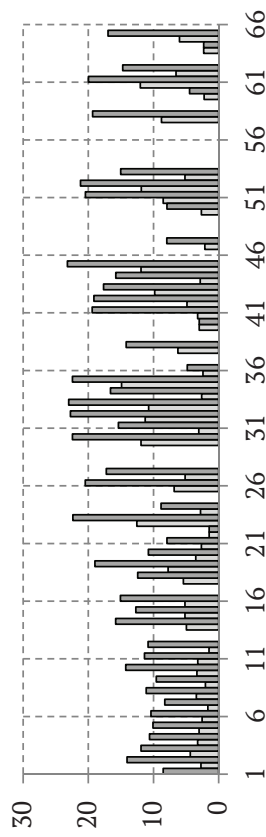


Fig. 3a. Changes in the sums of the weights of all concepts known to IIs

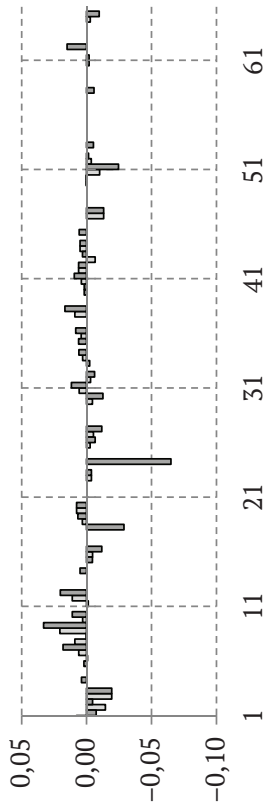


Fig. 3b. Changes in the levels of social engagement of IIs

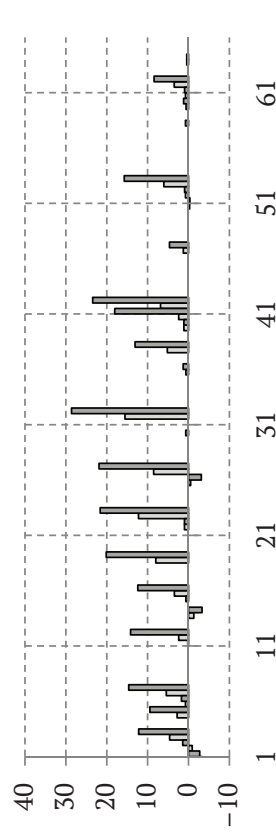


Fig. 3c. Changes in the sums of the weights of concepts to which IIs have positive attitudes

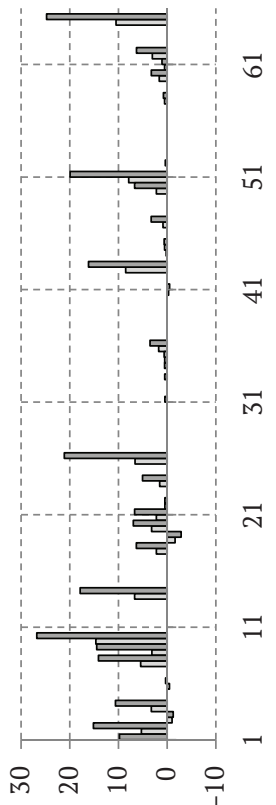


Fig. 3d. Changes in the sums of the weights of concepts to which IIs have negative attitudes

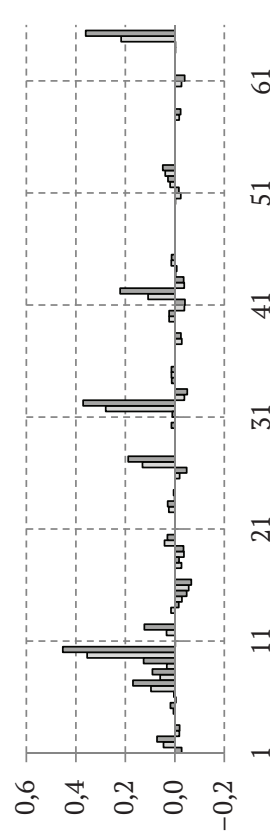


Fig. 3e. Changes in the differentiation of the personality structures of IIs

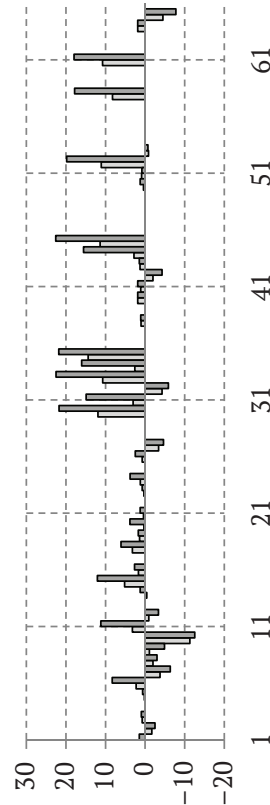


Fig. 3f. Changes in the sums of the weights of concepts to which IIs have indifferent attitudes

Fig. 3. Changes in the generalised characteristics of the personality structures of IIs that occurred during their adjustment in the third cycle of the interaction process against the background of changes that took place after the first cycle (dark columns correspond to the changes in the third cycle of contacts, light columns are the changes in the first cycle of contacts) (calculated by the author)

the weight of such concepts was a rare event). The average value was 3.6 SU within a range from  $-1.6$  to  $14.7$  SU. 21 out of 42 IIs changed their positive attitude. The average value was almost the same, 3.8 SU within a range from  $-3.3$  to  $15.5$  SU. After the third cycle of contacts, these figures were quite different. For a negative attitude to concepts, the average value was 8.0 SU within a range from  $-2.9$  to  $26.8$  (24 IIs) and for a positive attitude the average value was 8.9 SU within a range from  $-3.3$  to  $28.6$  SU (25 IIs out of 37).

As for the weights of concepts to which IIs were indifferent, almost all IIs that joined these groups changed it both in the first and in the second case (41 out of 42 IIs and 36 out of 37 IIs after the first and third cycles, respectively). The average value increased after the first cycle by 1.9 SU within a range from  $-11.3$  to  $14.4$  SU and after the third cycle it was 5.1 SU within a range from  $-12.5$  to  $22.6$  SU. More significant changes in the indifferent attitude to concepts as compared to the changes in the weights of positive and negative concepts can be explained by the fact that, as has been noted above, the ATM model assumes that when a new concept enters the thesaurus of an individual, their attitude to it and its weight are not always formed immediately. However, as a result of subsequent contacts, the indifferent opinion of the II about a concept under the influence of another II, or especially a group of IIs, might become quite certain and its weight might change. However, as we have mentioned before, there is another possibility that under a certain external influence the II might change their definite attitude to the concept for an indifferent one. Yet, such situations are less common in the ATM model.

As for changes in the degree of differentiation of the II's attitude to concepts, this indicator, which is a function of the ratio of concepts to which an II has a definite attitude, changed among all IIs belonging to these three groups and it was much more divergent than the changes in the weights of the corresponding concepts (Fig. 3e). Therefore, its average value after the first cycle of contacts was only 0.030 SU within a range from  $-0.056$  to  $0.355$  SU, and after the third cycle this average value was  $-0.052$  SU within a range from  $-0.066$  to  $0.45$  SU. The changes in social engagement, despite their variability,

were insignificant: from  $-0.019$  to  $0.021$  SU and the average value of 0 after the first cycle and from  $-0.025$  to  $0.033$  and the average value of 0 after the third cycle of contacts (Fig. 3b).

## Discussion

The first thing that should be mentioned is the fact that groups of individuals do emerge in a random sample of IIs. This means that even in a crowd of strangers who gathered to watch a fire, a fireworks display, or another public event there are always groups of individuals who will be of interest to each other due to similar ideas about the world. This is a well-known phenomenon when groups of people who find themselves together for a variety of reasons, for example, employees of a laboratory, members of a sports team or a work team or fans of an actor or a performer, etc., form interest groups (as sociologists call them) within these public groups (Andreeva, 2021).

Nevertheless, the ATM model showed that even in a random sample of individuals not belonging to a social group, groups of IIs were formed who were close to each other in their attitudes to the phenomena of the surrounding world. However, these groups differed from each other to a significant degree and had absolutely opposite personality structures of IIs belonging to these groups. Moreover, it was shown that repeated contacts between individuals within this group increase their confidence about their attitude to certain phenomena of the surrounding world, which is expressed in an increase in the weights of those concepts on which there is a consensus among the IIs within the groups.

It was quite unexpected to discover that such groups did not have any individuals with small thesauruses. Indeed, a priori it can be understood that if most of the concepts in the personality structure of an II coincide with a part of the personality structure of another II with a large or medium thesaurus, the first individual should come into contact with the second or join a group of IIs. However, it appears that such a connection is less strong than a connection between IIs with larger thesauruses and with more similarities in their personality structures. As a result, IIs with small thesauruses appear to be marginal members of this community.



It is important to note that the introduction of a random data generator into the algorithm of the ATM model allowed creating a fairly plausible picture of the mutual influence of IIs on each other since certain ideas or knowledge of one of the individuals do not always change the personality structure of the other individual. For example, this can happen due to a short interaction, the influence of the circumstances or simply because one II is not very eager to share ideas about a certain concept with another individual at a given time or in a given situation.

We would like to note in the conclusion that this research is important since such interpersonal, more or less random, but repeated and more frequent contacts are now largely facilitated by various Internet platforms, where two or more individuals do not need to be in a crowd of people or at a public meeting to meet each other for the first time.

### Conclusion

As we have mentioned above, the results obtained in this study allow analysing the emergence of groups of individuals and changes in their personality structures at the operational level by changing the parameters of the ATM model. A large number of parameters, which are included in this model, makes it possible to study the processes of how groups of individuals are formed at different distributions of the amounts of their thesauruses, proportions of positive, negative, and indifferent attitudes to concepts, levels of social engagement, and other, more local parameters.

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However, the ATM model does not explicitly contain such important elements of the process of group formation as emotions and other non-verbal phenomena associated with human nature and involved in the process of communication. Therefore, this model only considers the information component of the process of communication between individuals, or, as it is defined in the works dedicated to the theory of social groups, the communication component of this process (Sokolov, 2002), without taking into account the psychological characteristics of the group members.

It should be noted, however, that the absence of such elements in the model is largely compensated by the introduction of such a parameter as the social engagement of the individual. In particular, it can be interpreted as the position of the individual on the scale of "introvert – ambivert – extrovert", or some similar scales. Without a doubt, the distribution of individuals on this scale cannot fully describe the psychology of an individual, and, especially, of a group as a whole. However, if we consider a social group in terms of its activities in the external world, we should recognise that in most cases common purpose and ideas of the group members about the external world lead to more stable social associations and to more purposeful and united actions than emotional contacts.

### Conflict of Interest

The author declares the absence of obvious and potential conflicts of interest related to the publication of this article.

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## Математические и инструментальные методы экономики

Научная статья

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## Динамика формирования социальных групп и коррекция характеристик индивидов в процессе их объединения в группы

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**Предмет.** В статье рассматривается агент-ориентированная модель знакомства двух и более индивидов и образования ими малой социальной группы по интересам. Предполагается, что в процессе этого знакомства они обмениваются информацией о тех или иных проявлениях окружающего их мира и об отношении к ним. Моделируется также процесс изменений значимости данных проявлений в их представлениях об окружающем мире в результате этого общения.

**Цель.** Определить возможность формирования групп по интересам в случайной совокупности индивидов и характер изменений их отношений к окружающему миру в результате нескольких последовательных контактов между ними в рамках парадигмы агент-ориентированных моделей.

**Результаты.** В процессе формирования групп индивидов из их случайной совокупности методом иерархического кластер-анализа выявлены устойчивые группы по интересам, участники которых на протяжении серии контактов между ними значительно увеличили круг своих представлений об окружающем их мире и уверенность в правильности своих отношений к его проявлениям.

**Выводы.** Показано, что даже в случайной совокупности индивидов могут возникнуть кардинально противоположные по своим характеристикам группы индивидов, участники каждой из которых близки друг другу по своим представлениям об окружающем их мире. Показано также, что при наличии неоднократных контактов между индивидами этих групп имеет место значимый рост их уверенности в своем отношении к тем или иным проявлениям окружающего мира. Отмечается, что важной площадкой для таких контактов является в настоящее время Интернет.

**Ключевые слова:** агент-ориентированные модели, социальные группы, метод главных компонент, структура личности, тезаурус.

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### Конфликт интересов

Автор декларирует отсутствие явных и потенциальных конфликтов интересов, связанных с публикацией настоящей статьи.

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